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BMC 2024 13

QUIZ 2 - ALGEBRA I - AUGUST-NOVEMBER 2024

The time allotted for this quiz is 45 minutes. Write your name and roll number on every page that you use as an answer sheet. Write clearly, legibly, logically and justify all your assertions.

Throughout this quiz, $r =$ last two digits of your roll number + 5. For example, if your roll number is BMC202435, then $r = 40$.

- (1) Find a basis for the null space and the image of the matrix

$$\begin{bmatrix} 1 & r & 0 & r+1 \\ 1 & 2 & r & -2 \\ 2 & 2r & 1 & 1 \end{bmatrix}$$

Verify that the rank-nullity theorem holds for this matrix.

- (2) In each of the following, justify your answers completely with either an example or an argument:

- (i) Are there 4×4 matrices A, B of ranks 3 and 1 respectively such that $\text{rank of } AB = 0$?
(ii) Are there 4×4 matrices A, B of ranks 3 and 1 respectively such that $\text{rank of } AB = 2$?

- (3) Let $V = \mathbb{R}^3$ and let W be the vector subspace of V spanned by

$$\begin{bmatrix} 1 \\ 2 \\ r \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ r \\ 3 \end{bmatrix}$$

Find the matrix A of reflection in the plane W by using a suitable coordinate system.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Bx = \underline{Ax}$$

$$\text{rank } A$$

$$ABx = y$$

$$Ax = v$$

$$\text{rank } AB \leq \min(\text{rank } A, \text{rank } B)$$