$$\begin{bmatrix} a_{11}-x & & \\ & a_{22}-x & \\ & & \ddots \end{bmatrix}$$

$\Pi(x+a_{ii}) + \det(A) - \Pi a_{ii}$

QUIZ 3 - ALGEBRA 1 - AUGUST-NOVEMBER 2024

The time alloted for this quiz is 45 minutes. Write your name and roll number on every page that you use as an answer sheet. Write clearly, legibly, logically and justify all your assertions.

(1) Let $x_1, x_2, \ldots, x_n$ be variables. Find the determinant

$$\begin{vmatrix} s_0 & s_1 & s_2 & \cdots & s_{n-1} \\ s_1 & s_2 & s_3 & \cdots & s_n \\ s_2 & s_3 & s_4 & \cdots & s_{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{n-1} & s_n & s_{n+1} & \cdots & s_{2n-2} \end{vmatrix}$$

where $s_k = x_1^k + x_2^k + \cdots + x_n^k$.

(2) In this question, the matrices are defined *over complex numbers*, i.e. *they have complex entries*. Note that matrices defined over real numbers can also be thought of as defined *over complex numbers*.

(i) Show that if A, B are $n \times n$ matrices (*over complex numbers*) such that $I - AB$ is invertible, then $I - BA$ is also invertible and write down a formula for this inverse.

(ii) Show that AB and BA have the same eigenvalues as a set. Say with justification whether they are also the same as a multiset.

(iii) **Extra Credit:** Give an example of a vector space V and operators A and B on V such that AB is not invertible, but BA is. (Suggestion: Do not attempt this before having solved the other questions.)

$$\begin{bmatrix} 13 & 9 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 12 & 2 \\ 9 & 5 \end{bmatrix}$$

$1\cancel{2}\times 12$

B2. Write your solution to B2 below.