QUIZ 4 - ALGEBRA I - AUGUST-NOVEMBER 2024

The time alloted for this quiz is 45 minutes. Write your name and roll number on every page that you use as an answer sheet. Write clearly, legibly, logically and justify all your assertions.

(1) Prove or disprove: if V is a euclidean space or a Hermitian space and $T : V \to V$ is a linear operator such that $\langle Tv, v \rangle = 0$ for all $v \in V$, then T is the zero operator.

(2) Find an orthonormal basis for the column space of the following matrix:

$$M = \begin{bmatrix} 3 & -7 & 8 \\ -1 & 5 & -2 \\ 1 & 1 & 1 \\ 3 & -5 & 1 \end{bmatrix}.$$

(3) Let $T : V \to V$ be a linear transformation on $V = \mathbb{R}^n$ such that its matrix in some basis is a real symmetric matrix. Prove that $V = \ker(T) \oplus \operatorname{im}(T)$. Show as a consequence that T is an orthogonal projection onto $\operatorname{im}(T)$ if and only if in addition to being symmetric, $A^2 = A$.