Algebra II - Quiz 1 - 30 Marks One Hour Date:

1. Let $V = \mathbb{R}^3$, $v_1 = (1,1,0)$, $v_2 = (0,1,1)$, $w_1 = (1,0,1)$ and $w_2 = (0,0,1)$. Prove or disprove: There exists a linear transformation $T : V \to V$ such that $ker\ T = < v_1, v_2 >$ and $Im\ T = < w_1, w_2 >$. (5 marks) R-N T · trivial

2. Prove or disprove: If λ is an eigen value of an $n \times n$ matrix A, then λ^2 is an eigen value of A^2. (5 marks) exist

3. Let $m_1, \cdots, m_n$ be n positive real numbers. Let $A = (a_{ij})_{1 \le i,j \le n}$ be an $n \times n$ matrix where $a_{ij} = m_i/m_j$. Find the rank of A. (5 marks)

4. Let $A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Let $W = \{\mathbf{x} \in \mathbb{R}^3 : A\mathbf{x} = B\mathbf{x}\}$. Find a basis for W. (5 marks) $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

5. Suppose F is a field and $|F| = \infty$. Let V be a vector space over F. Show that V cannot be written as a union of finitely many proper subspaces. Does the same statement hold true if $|F|$ is finite. (10 marks)

$Ax = \dots$

$Bx = \dots$

$x = -2z$

$y = 2z$