# CM 1 – Mid-Sem-Exam – 4 Oct 2024, 9:30 am to 12:30 pm

1. {25 marks} **Springs between walls.** Four identical springs and three identical masses lie between two walls (see Fig. 1). Find the normal modes.

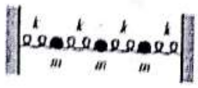



Figure 1: *Springs between walls. Find the normal modes.*

2. {25 marks} **Atwood's machine.** Consider the Atwood's machine in Fig. 2. It consists of three pulleys, a short piece of string connecting one mass to the bottom pulley, and a continuous long piece of string that wraps twice around the bottom side of the bottom pulley, and once around the top side of the top two pulleys. The two masses are $m$ and $2m$. Assume that the parts of the string connecting the pulleys are essentially vertical. Find the accelerations of the masses.

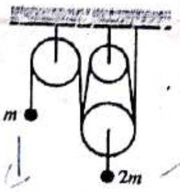



Figure 2: *Atwood's machine.*

3. {25 marks} **Colliding masses.** A mass $M$, initially moving at speed $V$, collides and sticks to a mass $m$, initially at rest. Assume $M \gg m$, and work in this approximation. What are the final energies of the two masses, and how much energy is lost to heat, in:

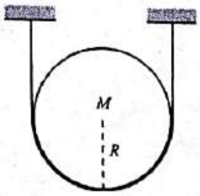
   (a) {12.5 marks} The lab frame?

   (b) {12.5 marks} The frame in which $M$ is initially at rest?

4. {25 marks} **Supporting a disk**

   (a) {12.5 marks} A disk of mass $M$ and radius $R$ is held up by a massless string, as shown in Fig. 3. The surface of the disk is frictionless. What is the tension in the string? What is the normal force per unit length that the string applies to the disk?

(b) {12.5 marks} Let there now be friction between the disk and the string, with coefficient $\mu$. What is the smallest possible tension in the string at its lowest point?




$\frac{mg}{2}\left(\frac{4\pi}{2+2\mu}+1\right)$

Figure 3: *Supporting a disk.*

$mg = \int_{-\pi/2}^{\pi/2} \rho R\, d\theta \cos\theta$

$\rho R \sin\theta \Big]_{-\pi/2}^{\pi/2}$

$T(\theta+d\theta)$, $T(\theta)$, $dN$

$\mu \rho R\, d\theta \sin\theta + \rho R\, d\theta \cos\theta$

$\mu \rho R \cos\theta \Big]_0^{\pi/2} + \rho R \sin\theta \Big]_0^{\pi/2}$ $\quad T(\theta+d\theta)\sin(\theta+d\theta)$

$\rho R - \mu \rho R$

$2\rho R(1-\mu) = mg$

$\rho = \frac{mg}{2R(1-\mu)}$