w
xyz

$a^k b^k c^k$
$uvxyz$

End-Semester Exam

CU2102 CG1103 Theory of Computation

Duration: 120 mins | Maximum Marks: 35

Notation: Let Σ be a finite alphabet. For a word $w \in \Sigma^*$ and a letter $a \in \Sigma$, we denote by $|w|_a$ the number of occurrences of the letter a in w.

1. Give a DFA[1] for the following language:

$$\{w \in \{a,b\}^* \mid |w|_a \text{ is even if, and only if, } |w|_b \text{ is even.}\}$$

(5 marks)

2. Let $\Sigma = \{a,b\}$. Give a one-counter automaton[1] for the following language.

$$\{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$$

(5 marks)

3. Let $\Sigma = \{a,b,c\}$. For $L_1 \subseteq \Sigma^*$ and $L_2 \subseteq \Sigma^*$, we define the following operators:

$$\bigcap_{\{a\},L_2}(L_1) = \{w \mid w \in L_1, \exists u \in L_2, |w|_a = |u|_a\}$$
$$\bigcap_{\{a,b\},L_2}(L_1) = \{w \mid w \in L_1, \exists u \in L_2, |w|_a = |u|_a, |w|_b = |u|_b\}$$
$$\bigcap_{\{a,b,c\},L_2}(L_1) = \{w \mid w \in L_1, \exists u \in L_2, |w|_a = |u|_a, |w|_b = |u|_b, |w|_c = |u|_c\}$$

$2K-1 > K$
$2K > K+1$
$K > 1$

$aabu$

(a) Describe how to construct an NFA for $\bigcap_{\{a\},L_2}(L_1)$ given a DFA for L_1 and a DFA for L_2. *(5 marks)*

(b) Prove that $\bigcap_{\{a,b\},L_2}(L_1)$ need not be regular even if L_1 and L_2 are regular. a^*b^* $|w|_a = |w|_b$ *(3 marks)*

(c) Describe how to construct a PDA for $\bigcap_{\{a,b\},L_2}(L_1)$ given a DFA for L_1 and a DFA for L_2. *(5 marks)* $\rightarrow |w|_a = |w|_b = |w|_c$

(d) Prove that $\bigcap_{\{a,b,c\},L_2}(L_1)$ need not be context-free even if L_1 and L_2 are regular. $a^n b^n c^n$ $a^*b^*c^*$ *(2 marks)*

4. Consider the following problem (All-Even).

Input: a context-free grammar G over $\{a,b\}$.
Question: Is $((a+b)(a+b))^* \subseteq L(G)$?

That is, All-Even asks if every even length word is generated by the input grammar G.

(a) Is All-Even recursively enumerable? Justify. *(5 marks)*

(b) Is All-Even co-recursively enumerable? Justify. *(5 marks)*

[1] Correctness proof is not needed.