

Mid-Semester Exam 2025

CU2102 CG1103 Theory of Computation

Duration: 120 mins

Maximum Marks: 40

1. Give a rational expression for the following language over $\Sigma = \{a, b\}$: $\{w \mid w \text{ contains at least four 'b's, or } w \text{ does not have consecutive 'b's}\}$. (3 marks)

2. Let $\Sigma = \{0, 1\}$. A word $w = b_0b_1 \dots b_{n-1}$ is seen as a binary number whose value is given by $\text{val}(w) \triangleq b_02^0 + b_12^1 + \dots + b_{n-1}2^{n-1}$. Note that the least significant bit is on the left. For a word w , w^R denotes its reverse (or mirror image). That is, if $w = b_0b_1 \dots b_{n-1}$, then $w^R = b_{n-1}b_{n-2} \dots b_1b_0$.

Give a pushdown automaton for the following language.

$$\{uv \mid u, v \in \{0, 1\}^*, \text{val}(u) \times 3 = \text{val}(v^R)\}.$$

(10 marks)

3. Let $u, v \in \Sigma^*$ be two words. We say that u is a *scattered subword* of v (denoted $u \preceq v$) if $u = a_1a_2 \dots a_n$ and $v = v_0a_1v_1a_2v_2 \dots a_nv_n$ for some $n \geq 0$ where $a_i \in \Sigma$ for $i \in \{1, 2, \dots, n\}$ and $v_i \in \Sigma^*$ for $i \in \{0, 1, \dots, n\}$. That is, u is obtained by "dropping" some letters from v .

Let $L \subseteq \Sigma^*$. Let $\min_{\preceq}(L) = \{w \mid w \in L, \forall u(u \in L \wedge u \preceq w) \Rightarrow u = w\}$. That is, it is the set of "minimal" words in L with respect to $\preceq$.

Prove that if L is regular then $\min_{\preceq}(L)$ is regular.

(6 marks)

4. Let $\Sigma = \{0, 1\}$. Recall that $\text{val}(w)$ gives the value of a binary number w with least significant bit on the left. Consider the following languages.

i. $L_1 = \{uv \mid \text{val}(u) + \text{val}(v) = 2^n \text{ for some } n \in \{0, 1, 2, \dots\}\}.$

ii. $L_2 = \{uv \mid \text{val}(u) + \text{val}(v) \neq 2^n \text{ for any } n \in \{0, 1, 2, \dots\}\}.$

iii. $L_3 = \{uv \mid \text{val}(u^R) + \text{val}(v) = 2^n \text{ for some } n \in \{0, 1, 2, \dots\}\}.$

- (a) Give a word in $L_1 \cap L_2$.

(1 marks)

- (b) For each of them say whether they are a) regular, b) not regular but context-free, or c) not context-free. Justify.

(20 marks)

(Hint: there is one of each case.)

For a) provide an NFA (no need for correctness proof); for b) provide a CFG/PDA (no need for correctness proof) and a proof of non-regularity; for c) provide a proof of non-context-freeness.