

Quiz 1

CU2102 CG1103 Theory of Computation

Duration: 100 mins

Maximum Marks: 20

1. Let $\Sigma = \{a, b\}$. Define $L = \{w \in \Sigma^* \mid |w|_a \equiv 0 \pmod{7}, \text{ or } |w|_b \equiv 0 \pmod{13}\}$.
 - (a) Given an NFA for L with 20 states. (1 marks)
 - (b) Describe a DFA for L with 91 states. (2 marks)
 - (c) Argue that it is impossible to get a DFA for L with less than 91 states. (4 marks)

2. For $L, K \subseteq \Sigma^*$ we define LK^{-1} as $\{u \mid \exists w \in K, uw \in L\}$. Describe how to obtain a finite state automaton for LK^{-1} , given DFA for L and K . (5 marks)

3. State whether the following statements are *true* or *false*. Argue why in a line or two. Here $K, L \subseteq \Sigma^*$ and $u, v \in \Sigma^*$. You can assume $\Sigma = \{a, b\}$. (4 marks)

(a) $u \cdot (u^{-1}L) = L$ **F**

(b) $K^{-1}(K \cdot L) = L$ **T**

(c) $(uv)^{-1}L = u^{-1}(v^{-1}L)$ **T**

(d) $u \equiv_L v$ if and only if $ua \equiv_L va$ and $ub \equiv_L vb$ **T**

(Here $\equiv_L$ denotes the Myhill-Nerode equivalence relation of L .)

4. Consider the context-free grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ where the productions P is given by:

$$S \rightarrow A \mid B \mid \epsilon$$

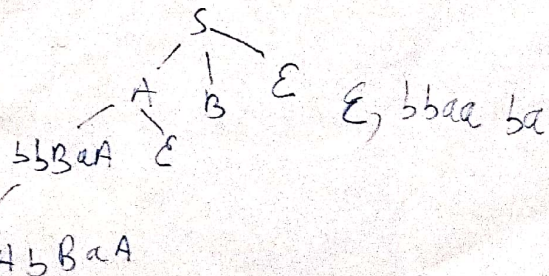
$$A \rightarrow bbBaA \mid \epsilon$$

$$B \rightarrow aaAbB \mid \epsilon$$

Is $L(G)$ regular? Justify.

(4 marks)

$\epsilon, bba, aab, aaabba, bbaaaba, \dots$



$$(bbaa)^i (ba)^j$$

$$\epsilon \leq i \iff i = j$$