Quiz 1

CU2102 CG1103 Theory of Computation

Duration: 100 mins — Maximum Marks: 20

1. Let $\Sigma = \{a, b\}$. Define $L = \{w \in \Sigma^* \mid |w|_a \equiv 0 \mod 7, \text{ or } |w|_b \equiv 0 \mod 13\}$.
   (a) Given an NFA for L with 20 states. *(1 marks)*
   (b) Describe a DFA for L with 91 states. *(2 marks)*
   (c) Argue that it is impossible to get a DFA for L with less than 91 states. *(4 marks)*

2. For $L, K \subseteq \Sigma^*$ we define LK^{-1} as $\{u \mid \exists w \in K, uw \in L\}$. Describe how to obtain a finite state automaton for LK^{-1}, given DFA for L and K. *(5 marks)*

3. State whether the following statements are *true* or *false*. Argue why in a line or two. Here $K, L \subseteq \Sigma^*$ and $u, v \in \Sigma^*$. You can assume $\Sigma = \{a, b\}$. *(4 marks)*
   (a) $u \cdot (u^{-1}L) = L$
   (b) $K^{-1}(K \cdot L) = L$
   (c) $(uv)^{-1}L = u^{-1}(v^{-1}L)$
   (d) $u \equiv_L v$ if and only if $ua \equiv_L va$ and $ub \equiv_L vb$
   (Here $\equiv_L$ denotes the Myhill-Nerode equivalence relation of L.)

4. Consider the context-free grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ where the productions P is given by:

$$S \to A \mid B \mid \epsilon$$
$$A \to bbBaA \mid \epsilon$$
$$B \to aaAbB \mid \epsilon$$

   Is $L(G)$ regular? Justify. *(4 marks)*