Integration Bee

Tournament Slides

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Tournament Format

- **Double Elimination**:
  - Players who lose once enter the **losing bracket**.
  - A loss in the losing bracket results in **elimination**.
  - The **winning bracket** and **losing bracket** each produce one **finalist**.
  - The **final round** is **single elimination**.
- If both the contestants fail to solve a question and one of the audience member solves it in the set time(without computational aids, obviously!), they will be awarded with the prize of one book! They can keep solving to win more than one book.

Matchup Rules

- Each matchup involves **2 contestants**.
- An **integral** is displayed simultaneously to both contestants, and a **timer starts**.
- Contestants must **circle or box** their answer to **lock it in**.
  - Once locked, answers **cannot be changed** until verified. They can continue working in the background if they wish.

Time Limits

- **Rounds 1 & 2**:
  - **Best of 2** questions.
  - **2 minutes** per question.
- **Rounds 3 & 4**:
  - **Best of 3** questions.
  - **5 minutes** per question.
- **Finals**:
  - **Best of 5** questions.
  - **5 minutes** per question.

Tie Breaks

- Indefinite Attempts will be provided.
- The first contestant to answer **correctly** within the time limit **advances**.

Aknowledgements

- The slides and paper was set by Piyush Kumar Jha(Qualification, Rd 1, Rd 3, Finals) and Arjun Maneesh Agarwal(Rd 2, Rd 4).
- The seminar hall setup as well as the stream setup was assisted by the CMI support staff.
- The streaming was handled by Arjun Maneesh Agarwal.
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- Finally, we thank all the participants and spectators for taking time off and joining us for CMI Integration Bee 2025.

Sponsors

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All thanks to them we were able to reward the participants suitably for their efforts.

Round 1

Question 1

$$\int_0^{\frac{\pi}{4}} \frac{\sin\theta - 2\ln\left(\frac{1-\sin\theta}{\cos\theta}\right)}{(1+\cos 2\theta)\sqrt{\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)}}\, d\theta$$

Answer 1

$$\sqrt{\ln(1+\sqrt{2})}$$

Question

$$\int_0^{\pi^2/4} \frac{dx}{1 + \sin\sqrt{x} + \cos\sqrt{x}}$$

Answer

$$\frac{\pi}{2} ln 2$$

Question

$$\int_0^{\frac{\pi}{2}} \sqrt{1 - 2\sin 2x + 3\cos^2 x}\; dx.$$

Answer

$$-3 + 2\sqrt{5}$$

Question

$$\int_0^{\pi} \frac{x^2\cos^2 x - x\sin x - \cos x - 1}{(1 + x\sin x)^2} dx$$

Answer

0

Question

$$\int \frac{2+\sqrt{x}}{(1+x+\sqrt{x})^2}dx$$

Answer

$$-2\frac{\sqrt{x}+1}{x+\sqrt{x}+1}+C.$$

Question

$$\int \frac{1 + x\cos x - \sin x}{\sqrt{x + \sin x + \cos x}\sqrt{(x - \sin x + \cos x)^3}}\, dx$$

Answer

$$\sqrt{\frac{x + \sin x + \cos x}{x - \sin x + \cos x}} + C$$

Question

$$\int \frac{1+\cos x-\sin x+x(\sin x+\cos x)}{(x+\sin x)(x+\cos x)}dx$$

Answer

$$\ln(x + \sin x) - \ln(x + \cos x) + C$$

Question

$$\int \frac{(x^2+1)(x^2+2)}{(x\cos x+\sin x)^4}\,dx$$

Answer

$$\tan\left(x + arctan\left(\frac{1}{x}\right)\right) + \frac{\tan^3\left(x + \operatorname{arctan}\left(\frac{1}{x}\right)\right)}{3} + C$$

Round 2

Question 1

$$\int \cos(x)\left(\ln(x) - \frac{1}{x^2}\right)\mathrm{d}x$$

Answer 1

$$\int \cos(x)\left(\ln(x) - \frac{1}{x^2}\right)\mathrm{d}x$$

$$= \boxed{\ln(x)\sin(x) + \frac{\cos(x)}{x} + C}$$

Question 2

$$\int_{-1}^{1} \frac{x^3+1}{x^2+1} (\arctan x)^2 \mathrm{d}x$$

Answer 2

$$\int_{-1}^{1} \frac{x^3+1}{x^2+1}(\arctan x)^2 \mathrm{d}x$$

$$= \boxed{\frac{\pi^3}{96}}$$

Question 3

$$\int_1^{e^2-2} \ln(x + \ln(x + \dots))\,\mathrm{d}x$$

Answer 3

$$\int_1^{e^2-2} \ln(x+\ln(x+\dots))\,\mathrm{d}x$$

$$= \boxed{e^2-1}$$

Question 4

$$\int_0^1 \frac{1-x}{\arcsin(x)\sqrt{1-x^2}-x^2+1}\mathrm{d}x$$

Answer 4

$$\int_0^1 \frac{1-x}{\arcsin(x)\sqrt{1-x^2}-x^2+1}\mathrm{d}x$$

$$= \boxed{\ln\left(\frac{\pi}{2}\right)}$$

Question 5

$$\int_0^{\frac{\pi}{2}} \sec^6(x) e^{-\tan(x)} \mathrm{d}x$$

Answer 5

$$\int_0^{\frac{\pi}{2}} \sec^6(x) e^{-\tan(x)} \mathrm{d}x$$

$$= \boxed{29}$$

Question 6

$$\int \frac{\mathrm{d}x}{\sqrt{2025^x - 1}}$$

Answer 6

$$\int \frac{\mathrm{d}x}{\sqrt{2025^x - 1}}$$

$$= \boxed{\frac{2}{\ln(2025)} \arctan(\sqrt{2025^x - 1})}$$

Question 7

$$\int (e^{3x} + e^{x})^2 (2e^{6x} + e^{4x}) \mathrm{d}x$$

Answer 7

$$\int (e^{3x} + e^{x})^2 (2e^{6x} + e^{4x}) \mathrm{d}x$$

$$= \boxed{\frac{1}{6} e^{6x} (e^{2x} + 1)^3}$$

Question 8

$$\int \frac{\mathrm{d}x}{(1+e^x)^2}$$

Answer 8

$$\int \frac{\mathrm{d}x}{(1+e^x)^2}$$

$$= \boxed{\ln\left(\frac{1}{e^{-x}+1}\right) - \frac{1}{e^{-x}+1}}$$

Round 3

Question 1

$$\sum_{n=0}^{2025} \int_0^1 \frac{dx}{2(x+n+1)\sqrt{(x+n)(x+n+1)}}$$

Answer 1

$$\sqrt{\frac{2026}{2027}}$$

Question 2

$$\int_0^1 (1+x+\cdots+x^{n-1})\{1+3x+\cdots+(2n-3)x^{n-2}+(2n-1)x^{n-1}\}dx.$$

(Note: $n \in \mathbb{N}$)

Answer 2

$$n^2$$

Question 3

$$\int_{\frac{\pi}{8}}^{\frac{3}{8}\pi} \frac{11 + 4\cos 2x + \cos 4x}{1 - \cos 4x}\, dx.$$

Answer 3

$$6 - \frac{\pi}{4}$$

Question 4

$$\int_0^{\frac{\pi}{2}} \cos^n x \sin(n+2)x \;\, dx$$

(Note: $n \in \mathbb{N}$)

Answer 4

$$\frac{1}{n+1}$$

Question 5

$$\int \frac{x\,dx}{(x^2+x+1)^{3/2}}$$

Answer 5

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}+C$$

Question 6

$$\int \frac{e^{2x} - e^{x} + 1}{(e^{x} \sin x + \cos x)(e^{x} \cos x - \sin x)} dx$$

Answer 6

$$\ln\left|\frac{e^x \sin x + \cos x}{e^x \cos x - \sin x}\right| + C$$

Question 7

$$\int \frac{e^x(x^4+2)dx}{(x^2+1)^{5/2}}$$

Answer 7

$$\frac{e^x(x^2+x+1)}{(1+x^2)^{\frac{3}{2}}}+C$$

Question 8

$$\int \frac{e^x(x^4+2)dx}{(x^2+1)^{5/2}}$$

Answer 8

$$\frac{e^x(x^2+x+1)}{(1+x^2)^{\frac{3}{2}}}+C$$

Question 9

$$\int_0^{\pi} \left(1 + \sum_{k=1}^{n} k\cos(kx)\right)^2 dx \ (n = 1, 2, \cdots).$$

Answer 9

$$\frac{\pi}{2}(1^2 + 2^2 + 3^2 + \ldots + n^2) + \pi \text{ Or}$$

$$\frac{\pi}{12}(2n^3 + 3n^2 + n + 12)$$

Round 4

Question 9

$$\int_0^1 \sum_{n=1}^{\infty} \frac{1}{n} \sum_{m=0}^{\infty} (-x)^{m+n} \mathrm{d}x$$

Answer 9

$$\int_0^1 \sum_{n=1}^{\infty} \frac{1}{n} \sum_{m=0}^{\infty} (-x)^{m+n} \mathrm{d}x$$

$$= \boxed{\frac{\ln^2(2)}{2}}$$

Question 10

$$\int_0^{\pi/4} ln(\cot(x) - 1)\,\mathrm{d}x$$

Answer 10

$$\int_0^{\pi/4} ln(\cot(x) - 1)\mathrm{d}x$$

$$= \boxed{\ln(2)\frac{\pi}{8}}$$

Question 11

$$\int \frac{\tan x \sec x - \tan x \sin x + \sec x - \sin x}{1 + \sin 2x} \mathrm{d}x$$

Answer 11

$$\int \frac{\tan x \sec x - \tan x \sin x + \sec x - \sin x}{1 + \sin 2x} \mathrm{d}x$$

$$= \boxed{\frac{\tan x}{\sin x + \cos x}}$$

Question 12

$$\int_0^1 \operatorname{arcsin}\left(\sqrt{1-\sqrt{x}}\right) \mathrm{d}x$$

Answer 12

$$\int_0^1 \arcsin\left(\sqrt{1-\sqrt{x}}\right)\mathrm{d}x$$

$$= \boxed{\frac{3\pi}{16}}$$

Question 13

$$\int_0^\infty \left(\frac{1}{x^{x+1}}\right)\left(1-\frac{e^{-xe}}{(x^x)^{e-1}}\right)\mathrm{d}x$$

Answer 13

$$\int_0^{\infty} \left(\frac{1}{x^{x+1}}\right)\left(1 - \frac{e^{-xe}}{(x^x)^{e-1}}\right) \mathrm{d}x$$

$$= \boxed{1}$$

Question 14

$$\int_0^{2\pi} \left(\sum_{i=1}^{n} \sin(ix) \right)^2 \, \mathrm{d}x$$

Answer 14

$$\int_0^{2\pi} \left(\sum_{i=1}^{n} \sin(ix) \right)^2 \mathrm{d}x$$

$$= \boxed{n\pi}$$

Question 15

$$\int \frac{\tan 2x}{\tan^2 x} \mathrm{d}x$$

Answer 15

$$\int \frac{\tan 2x}{\tan^2 x} \mathrm{d}x$$

$$= \boxed{2\log(\sin x) - \frac{1}{2}\log(\cos x) + C}$$

Question 16

$$\int_1^\infty \frac{x^{2024} \ln x}{(x^{2025}+1)^2} \mathrm{d}x$$

Answer 16

$$\int_1^\infty \frac{x^{2024}\ln x}{(x^{2025}+1)^2}\mathrm{d}x$$

$$= \boxed{\ln 2(\ln 2025 + 1)}$$

Question 17

$$\int \ln(x)\ln(1-\ln(x))\mathrm{d}x$$

Answer 17

$$\int \ln(x)\ln(1-\ln(x))\mathrm{d}x$$

$$= \boxed{x\ln(1-\ln x)(\ln x - 1) + C}$$

Round 5

Question 1

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos x + 1 - x^2}{(1 + x\sin x)\sqrt{1 - x^2}} dx$$

Answer 1

$$2\arcsin\left(\frac{4+\pi\sqrt{2}}{\pi+4\sqrt{2}}\right)$$

Question 2

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{1+\cos^2 x} dx$$

Answer 2

$$\frac{\pi}{4}$$

Question 3

$$\int_1^3 \left(\arctan\left(\frac{x^2-5x+6}{x^3-6x^2+12x-7}\right)+\arctan\left(\frac{1}{x^2-4x+4}\right)\right)dx$$

Answer 3

$$\pi$$

Question 4

$$\int_0^\infty \frac{e^{-x}(1-\cos(3x))}{x^2}dx$$

Answer 4

$$3\arctan(3) - \ln(\sqrt{10})$$

Question 5

$$\int \frac{\arctan^2(x)}{(x - \arctan^2(x))^2} dx$$

Answer 5

$$\frac{x\arctan(x)+1}{\arctan(x)-x}+C$$

Tie-Breakers

Question 1

$$\int_1^2 (x-1)^{1/2}(2-x)^{1/2}dx$$

Answer 1

$$\boxed{\frac{\pi}{8}}$$

Question 2

$$\int_0^{\pi/2} \frac{dx}{\sin(x) + \sec(x)}$$

Answer 2

$$\boxed{\frac{\log(2+\sqrt{3})}{\sqrt{3}}}$$

Question 3

$$\int_0^\infty \frac{dx}{\sqrt{1 + e^x + e^{2x}}}$$

Answer 3

$$\boxed{\log\left(\frac{3+2\sqrt{3}}{3}\right)}$$

Question 4

$$\int \frac{\cos(x) + x\sin(x)}{x(x+\cos(x))}dx$$

Answer 4

$$\boxed{\log\left(\frac{x}{x+\cos(x)}\right)+C}$$

Question 5

$$\int_0^1 \sin^2(\log(x))dx$$

Answer 5

$$\frac{2}{5}$$

Question 6

$$\int \sqrt[3]{3\sin(x) - \sin(3x)}\,dx$$

Answer 6

$$-\sqrt[3]{4}\cos(x) + C$$

Question 7

$$\int (\sec^4(x) - \tan^4(x))dx$$

Answer 7

$$2\tan(x) - x + C$$

Question 8

$$\int_0^{20} \left\lceil \frac{\lfloor x \rfloor}{2} \right\rceil dx$$

Answer 8

100

Question 9

$$\int_1^2 (2^{x-1} + \log_2(2x))dx.$$

Answer 9

3