Integration Bee Tournament Slides

Arjun Maneesh Agarwal Piyush Kumar Jha



Chennai Mathematical Institute

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Double Elimination:

- Players who lose once enter the losing bracket.
- A loss in the losing bracket results in **elimination**.
- The winning bracket and losing bracket each produce one finalist.
- The final round is single elimination.
- If both the contestants fail to solve a question and one of the audience member solves it in the set time(without computational aids, obviously!), they will be awarded with the prize of one book! They can keep solving to win more than one book.

Matchup Rules



- Each matchup involves **2** contestants.
- An integral is displayed simultaneously to both contestants, and a timer starts.
- Contestants must circle or box their answer to lock it in.
 - Once locked, answers cannot be changed until verified. They can continue working in the background if they wish.

Time Limits



Rounds 1 & 2:

- Best of 2 questions.
- 2 minutes per question.
- Rounds 3 & 4:
 - Best of 3 questions.
 - **5 minutes** per question.
- Finals:
 - Best of 5 questions.
 - **5 minutes** per question.

Tie Breaks



- Indefinite Attempts will be provided.
- The first contestant to answer correctly within the time limit advances.

Aknowledgements



- The slides and paper was set by Piyush Kumar Jha(Qualification, Rd 1, Rd 3, Finals) and Arjun Maneesh Agarwal(Rd 2, Rd 4).
- The seminar hall setup as well as the stream setup was assisted by the CMI support staff.
- ► The streaming was handled by Arjun Maneesh Agarwal.
- Special Thanks to Vignesh Sangle for Handling Tessalate 2025 and providing us the opportunity, Aditi Mishra for helping with paper printing and exam setup and Anusha Gupta for photography.
- Finally, we thank all the participants and spectators for taking time off and joining us for CMI Integration Bee 2025.



The books awarded to the participants and winners were donated by

- Prof. SP Suresh
- Prof. HS Mani
- Prof. Sukhendu Mehrotra
- Prof. Priyavrat C Deshpande
- Prof. Rajeeva L Karandiker
- Prof. Amitabh Virmani
- Prof. Govind Krishnaswammi
- Prof. Ghanshyam Date
- Rajeshwari Nair Ma'am

All thanks to them we were able to reward the participants suitably for their efforts.





$$\int_{0}^{\frac{\pi}{4}} \frac{\sin \theta - 2 \ln \left(\frac{1 - \sin \theta}{\cos \theta}\right)}{(1 + \cos 2\theta) \sqrt{\ln \left(\frac{1 + \sin \theta}{\cos \theta}\right)}} \, d\theta$$



 $\sqrt{\ln(1+\sqrt{2})}$





$$\int_0^{\pi^2/4} \frac{dx}{1+\sin\sqrt{x}+\cos\sqrt{x}}$$





 $\frac{\pi}{2}$ In2

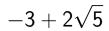




$$\int_0^{\frac{\pi}{2}} \sqrt{1 - 2\sin 2x + 3\cos^2 x} \, dx.$$











$$\int_0^{\pi} \frac{x^2 \cos^2 x - x \sin x - \cos x - 1}{(1 + x \sin x)^2} dx$$





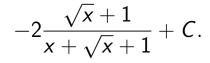




 $\int \frac{2 + \sqrt{x}}{(1 + x + \sqrt{x})^2} dx$











$$\int \frac{1 + x \cos x - \sin x}{\sqrt{x + \sin x + \cos x} \sqrt{(x - \sin x + \cos x)^3}} \, dx$$



$$\sqrt{\frac{x+\sin x+\cos x}{x-\sin x+\cos x}}+C$$





$$\int \frac{1 + \cos x - \sin x + x(\sin x + \cos x)}{(x + \sin x)(x + \cos x)} dx$$





$\ln(x+\sin x) - \ln(x+\cos x) + C$





 $\int \frac{(x^2+1)(x^2+2)}{(x\cos x+\sin x)^4} dx$



$$\tan\left(x+\arctan\left(\frac{1}{x}\right)\right)+\frac{\tan^3\left(x+\arctan\left(\frac{1}{x}\right)\right)}{3}+C$$





$$\int \cos(x) \left(\ln(x) - \frac{1}{x^2} \right) \mathrm{d}x$$

Answer 1



$$\int \cos(x) \left(\ln(x) - \frac{1}{x^2} \right) dx$$
$$= \left[\ln(x) \sin(x) + \frac{\cos(x)}{x} + C \right]$$



$$\int_{-1}^{1} \frac{x^3 + 1}{x^2 + 1} (\arctan x)^2 \mathrm{d}x$$



$$\int_{-1}^{1} \frac{x^3 + 1}{x^2 + 1} (\arctan x)^2 \mathrm{d}x$$
$$= \boxed{\frac{\pi^3}{96}}$$



$$\int_1^{e^2-2} \ln(x+\ln(x+\dots)) \mathrm{d}x$$

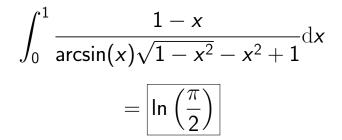


$$\int_{1}^{e^{2}-2} \ln(x + \ln(x + \dots)) dx$$
$$= \boxed{e^{2}-1}$$



$$\int_0^1 \frac{1-x}{\arcsin(x)\sqrt{1-x^2}-x^2+1} \mathrm{d}x$$







$$\int_0^{\frac{\pi}{2}} \sec^6(x) e^{-\tan(x)} \mathrm{d}x$$

Answer 5



$$\int_0^{\frac{\pi}{2}} \sec^6(x) e^{-\tan(x)} \mathrm{d}x$$
$$= \boxed{29}$$



 $\int \frac{\mathrm{d}x}{\sqrt{2025^x - 1}}$



$$\int \frac{\mathrm{d}x}{\sqrt{2025^{\mathsf{x}}-1}}$$

$$= \boxed{\frac{2}{\ln(2025)} \arctan(\sqrt{2025^{\times}-1})}$$





$$\int (e^{3x} + e^x)^2 (2e^{6x} + e^{4x}) \mathrm{d}x$$



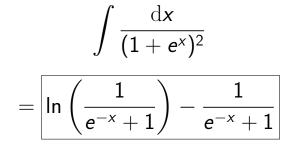
$$egin{aligned} &\int (e^{3x}+e^x)^2(2e^{6x}+e^{4x})\mathrm{d}x \ &= \overline{\left[rac{1}{6}e^{6x}(e^{2x}+1)^3
ight]} \end{aligned}$$

Question 8



$\int \frac{\mathrm{d}x}{(1+e^x)^2}$









$$\sum_{n=0}^{2025} \int_0^1 \frac{dx}{2(x+n+1)\sqrt{(x+n)(x+n+1)}}$$



$\sqrt{\frac{2026}{2027}}$



$$\int_0^1 (1+x+\cdots+x^{n-1})\{1+3x+\cdots+(2n-3)x^{n-2}+(2n-1)x^{n-1}\}dx.$$

(Note: $n \in \mathbb{N}$)





n²



$$\int_{\frac{\pi}{8}}^{\frac{3}{8}\pi} \frac{11 + 4\cos 2x + \cos 4x}{1 - \cos 4x} \, dx.$$



 $6-rac{\pi}{4}$



$$\int_0^{\frac{\pi}{2}} \cos^n x \sin(n+2)x \, dx$$
(Note: $n \in \mathbb{N}$)



$\frac{1}{n+1}$



$$\int \frac{x\,dx}{(x^2+x+1)^{3/2}}$$



$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}+C$



$$\int \frac{e^{2x} - e^x + 1}{(e^x \sin x + \cos x)(e^x \cos x - \sin x)} dx$$



$$\ln \left| \frac{e^x \sin x + \cos x}{e^x \cos x - \sin x} \right| + C$$





 $\int \frac{e^{x}(x^{4}+2)dx}{(x^{2}+1)^{5/2}}$



$$\frac{e^{x}(x^{2}+x+1)}{(1+x^{2})^{\frac{3}{2}}}+C$$



 $\int \frac{e^{x}(x^{4}+2)dx}{(x^{2}+1)^{5/2}}$



$$\frac{e^{x}(x^{2}+x+1)}{(1+x^{2})^{\frac{3}{2}}}+C$$



$$\int_0^{\pi} \left(1 + \sum_{k=1}^n k \cos(kx) \right)^2 dx \ (n = 1, 2, \cdots).$$



$$\frac{\pi}{2}(1^2 + 2^2 + 3^2 + \dots + n^2) + \pi \operatorname{Or}$$
$$\frac{\pi}{12}(2n^3 + 3n^2 + n + 12)$$





 $\int_0^1 \sum_{n=1}^\infty \frac{1}{n} \sum_{m=0}^\infty (-x)^{m+n} \mathrm{d}x$



$$\int_0^1 \sum_{n=1}^\infty \frac{1}{n} \sum_{m=0}^\infty (-x)^{m+n} \mathrm{d}x$$
$$= \boxed{\frac{\ln^2(2)}{2}}$$



$$\int_0^{\pi/4} \ln(\cot(x) - 1) \mathrm{d}x$$



$$\int_0^{\pi/4} \ln(\cot(x) - 1) \mathrm{d}x$$
$$= \boxed{\ln(2)\frac{\pi}{8}}$$



$\int \frac{\tan x \sec x - \tan x \sin x + \sec x - \sin x}{1 + \sin 2x} dx$



$$\int \frac{\tan x \sec x - \tan x \sin x + \sec x - \sin x}{1 + \sin 2x} dx$$
$$= \frac{\tan x}{\sin x + \cos x}$$



$$\int_0^1 \arcsin\left(\sqrt{1-\sqrt{x}}\right) \mathrm{d}x$$



$$\int_{0}^{1} \arcsin\left(\sqrt{1-\sqrt{x}}\right) \mathrm{d}x$$
$$= \boxed{\frac{3\pi}{16}}$$



$\int_0^\infty \left(\frac{1}{x^{x+1}}\right) \left(1 - \frac{e^{-xe}}{(x^x)^{e-1}}\right) \mathrm{d}x$



 $\int_0^\infty \left(\frac{1}{x^{x+1}}\right) \left(1 - \frac{e^{-xe}}{(x^x)^{e-1}}\right) \mathrm{d}x$ = |1|



 $\int_0^{2\pi} \left(\sum_{i=1}^n \sin(ix)\right)^2 \mathrm{d}x$



$$\int_{0}^{2\pi} \left(\sum_{i=1}^{n} \sin(ix) \right)^{2} dx$$
$$= \boxed{n\pi}$$



$\int \frac{\tan 2x}{\tan^2 x} \mathrm{d}x$



$$\int \frac{\tan 2x}{\tan^2 x} dx$$
$$= 2\log(\sin x) - \frac{1}{2}\log(\cos x) + C$$



 $\int_{1}^{\infty} \frac{x^{2024} \ln x}{(x^{2025} + 1)^2} \mathrm{d}x$



$$\int_{1}^{\infty} \frac{x^{2024} \ln x}{(x^{2025} + 1)^2} dx$$
$$= \boxed{\ln 2(\ln 2025 + 1)}$$



$\int \ln(x) \ln(1 - \ln(x)) \mathrm{d}x$



$$\int \ln(x) \ln(1 - \ln(x)) dx$$
$$= \boxed{x \ln(1 - \ln x)(\ln x - 1) + C}$$





$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos x + 1 - x^2}{(1 + x \sin x)\sqrt{1 - x^2}} dx$$



$$2 \arcsin\left(\frac{4+\pi\sqrt{2}}{\pi+4\sqrt{2}}\right)$$





$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{1 + \cos^2 x} dx$$





$\frac{\pi}{4}$



$$\int_{1}^{3} \left(\arctan\left(\frac{x^2 - 5x + 6}{x^3 - 6x^2 + 12x - 7}\right) + \arctan\left(\frac{1}{x^2 - 4x + 4}\right) \right) dx$$



 π





$$\int_0^\infty \frac{e^{-x}(1-\cos(3x))}{x^2}dx$$



$3 \arctan(3) - \ln(\sqrt{10})$





$$\int \frac{\arctan^2(x)}{(x - \arctan^2(x))^2} dx$$



$\frac{x\arctan(x)+1}{\arctan(x)-x}+C$

Tie-Breakers



$$\int_{1}^{2} (x-1)^{1/2} (2-x)^{1/2} dx$$

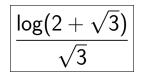






$$\int_0^{\pi/2} \frac{dx}{\sin(x) + \sec(x)}$$

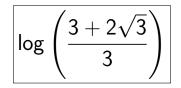






$$\int_0^\infty \frac{dx}{\sqrt{1+e^x+e^{2x}}}$$







 $\int \frac{\cos(x) + x\sin(x)}{x(x + \cos(x))} dx$



$$\log\left(\frac{x}{x+\cos(x)}\right)+C$$



 $\int_0^1 \sin^2(\log(x)) dx$





$$\int \sqrt[3]{3\sin(x) - \sin(3x)} dx$$



$-\sqrt[3]{4}\cos(x)+C$





 $\int (\sec^4(x) - \tan^4(x)) dx$



$2\tan(x) - x + C$





$\int_0^{20} \lceil \frac{\lfloor x \rfloor}{2} \rceil dx$



100



$$\int_{1}^{2} (2^{x-1} + \log_2(2x)) dx.$$



3